Use of Finite Element Analysis to Predict Laser-generated Lamb Waves

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ABSTRACT

Lamb waves that are usually generated in thin structures such as plates and pipes are widely used for nondestructive testing and characterization of materials. However, in the case of laser-generated Lamb waves, experimental observations have some limitations with respect to both sensitivity and accuracy in Lamb wave detection. From a thermo-elastic viewpoint, an approach that represents three-dimensional geometry of thin structures and describes thermal diffusivity into subsurface due to laser heat source may provide improved understanding of material behavior. In this regard, we employ finite element analysis (FEA) to predict Lamb waves induced by laser pulse and to evaluate its potential for the purpose of structural health monitoring. This analysis gives a systematic process and allows one to investigate thermo-elastic generation of longitudinal, shear, and surface waves as well as guided Lamb wave modes in a thin plate.

INTRODUCTION

In the wake of severe disasters such as I-35W Bridge and Seongsu Bridge collapse, many civil engineers have intensified their efforts to develop effective techniques for detecting damages in structures (Lim et al. 2013). In the inspection of damages in a broad range of structures and materials from civil infrastructure and aerospace industry, nondestructive testing has been emerged as an effective tool. Among the techniques of nondestructive testing, ultrasound based method is especially suitable for a testing of thin structures (e.g. plates and pipes). In ultrasonic testing, elastic waves are generated on the surface of structures, and then propagated along the media. The robustness of the structures is investigated thoroughly by analyzing the response of the waves.

Ultrasonic testing can be classified into two major categories according to the way of generating waves. One is the method using piezoelectric transducers for producing elastic waves. In the testing by using piezoelectric transducers, toneburst signals are employed; investigators does not confused in processing the signals obtained because the narrow bandwidth of the signals can help one to analyze the response without regarding the influence of dispersion. Another method is laser-generated ultrasound. Unlike the piezoelectric transducers, laser can produce broadband signals. It is often desirable to utilize the broadband signals in nondestructive testing because information over the wide range of frequency can be useful especially in the characterization of materials.

Laser can generate bulk waves such as longitudinal waves (P-waves) and transverse waves (S-waves), and also guided waves like Rayleigh waves (surface waves) and Lamb waves (Giurgiutiu 2008). The generation of different types of waves is mainly depends on the geometry of the specimen. Bulk waves including longitudinal and transverse waves are induced by thermal diffusivity or optical penetration by a pulse laser. Rayleigh waves and Lamb waves are product as the result of the interaction of longitudinal and transverse waves in the boundary of the specimen. Among many wave modes, of particular interests are the generation of Lamb waves due to its availability and flexibility.

Many efforts have been made towards the researches of laser-generated Lamb waves both experimentally and analytically (Lie et al. 2012). In the experiment using laser, Nd:YAG (neodymium-doped yttrium aluminum garnet) laser and laser Doppler vibrometer are frequently used (Pierce et al. 1997). Also, optical fiber sensors connected to a fiber interferometer is used for a particular purpose which means the route of laser is not straight. By contrast, small numbers of studies are found that are closely related to laser-generated Lamb waves. In most of the works about solving laser-generated Lamb waves analytically, governing equation is solved by taking integral transformation or Green’s function formalism.

This paper is devoted to the development of numerical and analytical model of laser-generated Lamb waves. In the numerical model, the finite element analysis program (FEAP) is employed to simulate the transient behavior of Lamb waves. To predict accurate response of Lamb waves, multiscale modeling analysis including microscale and macroscale models are

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proposed. In the following, the procedure to solve the thermo-elastic equation using Fourier and Hankel transformation method is proposed.

THEORY

Finite element formulation for Lamb wave propagation

The numerical method used in present paper follows common procedure of well-known finite element formulation for thermoelasticity. One important underlying characteristic of thermoelasticity is that the heat energy from the pulse laser corresponds to the source of generation of Lamb waves. After the shot by the pulse laser, thermal expansion on the near-surface in the specimen causes the propagation of thermoelastic waves, i.e., laser-generated Lamb waves. Here, brief summarized finite element equations in the (Liu 2012) for thermoelasticity are stated.

For numerical implementation, thermomechanically coupled equations are given as

\[ \mathbf{M}\ddot{\mathbf{U}} + K_{uu}\Theta + K_{u\theta}\mathbf{U} = \mathbf{F} \]  \hspace{1cm} (1)

\[ C_{\partial\theta}\ddot{U} + C_{\partial\theta}\Theta + K_{\partial\theta}\Theta = \mathbf{Q} \]  \hspace{1cm} (2)

where \( K_{uu} = \int_V \mathbf{B}_u^T \mathbf{C} \mathbf{B}_u dV \), \( K_{u\theta} = \int_V \mathbf{B}_u^T \beta \mathbf{H}_\theta dV \), \( K_{\partial\theta} = \int_V \mathbf{B}_{\partial\theta}^T \mathbf{B}_{\partial\theta} dV \), \( \mathbf{M} = \int_V \mathbf{H}_u^T \rho \mathbf{H}_u dV \), \( C_{\partial\theta} = T_0 \int_V \mathbf{H}_{\partial\theta}^T \beta \mathbf{B}_{\partial\theta} dV \), \( C_{\partial\theta} = \int_V \mathbf{H}_{\partial\theta}^T c \mathbf{H}_{\partial\theta} dV \), \( \mathbf{F} = \int_I \mathbf{H}_u^T \mathbf{f} dA + \int_V \mathbf{H}_u^T \mathbf{f} dV \), \( \mathbf{Q} = -\int_I \mathbf{H}_{\partial\theta}^T \mathbf{Q} dA \).

Analytical solution of transient response due to laser source

Among the analytical methods for solving the governing equation, the method using Fourier and Hankel transformation is used in this paper. Procedures to deal with the problem fall into three steps: problem statement, Fourier and Hankel transformation, and residue theorem. Products about the steps are summarized in this chapter.

Problem statement

A cylindrical coordinate system is considered in the problem as shown in Fig. 1, where an isotropic plate of thickness \( h \) is subjected to a normal load \( f(r,t) \) distributed in a circular area. The stress boundary conditions are given as

\[ \sigma_{zz}(r,t) = \begin{cases} f(r,t) & \text{at } z = +h \\ 0 & \text{at } z = -h \end{cases} \]
\[ \sigma_{r\theta}(r,t) = 0 & \text{at } z = \pm h. \]  \hspace{1cm} (3)

The equations of motion with the condition \( u_\theta = \partial \sigma / \partial \theta = 0 \) are stated as

\[ (\lambda + 2\mu) \frac{\partial^2 \Delta}{\partial r^2} + \mu \frac{\partial^2 \Delta}{\partial z^2} = \rho \frac{\partial^2 u_r}{\partial t^2} \]
\[ (\lambda + 2\mu) \frac{\partial \Delta}{\partial z} - \frac{\mu}{r} \frac{\partial (r\Omega)}{\partial r} = \rho \frac{\partial^2 u_r}{\partial t^2}. \]  \hspace{1cm} (4)
Fourier and Hankel transformation  Our interests is to get the transformed displacement fields caused by the distributed normal force. To obtain the displacement fields, double integral transformation scheme is employed (Graff 1975). Hankel transformation is applied to the spatial domain while in the time domain Fourier transformation is applied. After applying the transformations, the transformed displacements is defined as (Shi et al. 2003)

$$
\hat{u}_z(z,k,\omega) = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u_z(z,r,t) r J_0(kr)e^{j\omega t} dr dt
$$

and

$$
\hat{u}_r(z,k,\omega) = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u_r(z,r,t) r J_1(kr)e^{j\omega t} dr dt
$$

where $\hat{u}_z$ is the out-of-plane displacement field, $\hat{u}_r$ is the in-plane displacement field, $J_0(kr)$ and $J_1(kr)$ are the Bessel functions of the zeroth and first-order, respectively.

Residue theorem  Since the transformed displacement fields can contain the functions that have an infinitely large number of poles, it is mandatory to use the residue theorem to calculate integral over the wavenumber $k$. By using the residue theorem, one can get the surface displacement fields expressed as (Shi et al. 2003)

$$
u_z(h,x,t) = \int_{-\infty}^{+\infty} \sum_{k_z} H_z^s(h,\omega) & k J_0(kr)e^{-j\omega t} \omega + \int_{-\infty}^{+\infty} \sum_{k_a} H_a^a(h,\omega) & k J_0(kr)e^{-j\omega t} \omega \nabla
$$

where $H_z^s(h,\omega)$ and $H_a^a(h,\omega)$ are the material responses expressed as

$$
H_z^s(h,\omega) = -\frac{j\alpha(k_z^2 - \beta^2)^2 \sinh(\alpha h) \sinh(\beta h)}{4\mu\Delta_a},
$$

$$
H_a^a(h,\omega) = -\frac{j\alpha(k_a^2 - \beta^2)^2 \cosh(\alpha h) \cosh(\beta h)}{4\mu\Delta_a}.
$$

where

$$
\Delta_a = (k^2 + \beta^2)^2 \cosh(\alpha h) \sinh(\beta h) - 4k^2\alpha\beta \sinh(\alpha h) \cosh(\beta h)
$$

$$
\Delta_a = (k^2 + \beta^2)^2 \sinh(\alpha h) \cosh(\beta h) - 4k^2\alpha\beta \cosh(\alpha h) \sinh(\beta h).
$$

Here $\alpha^2 = k^2 - \omega^2/c_L^2$, $\beta^2 = k^2 - \omega^2/c_T^2$, $c_L$ is the longitudinal wave speed and $c_T$ is the transverse wave speed.

**NUMERICAL RESULTS**

**Multiscale modeling**

Thermal wave propagation induced by ultrashort laser pulses occurs in the plate at its several structure levels due to the presence of the certain geometric and the physical entities in regard to the ultrasound generation. In order to obtain the comprehensive understanding of the ultrasound generation, one should get an insight on how ultrasound generation mechanisms interact with each other. In this section, details about mechanisms of thermomechanical elastic wave propagation are stated for the process of multiscale modeling.

A bottom-up analysis for a multiscale modeling is proposed to predict thermomechanical wave propagation in plate in the aspects of the two different scales, the microscale and the macroscale, as shown in Fig. 2. In this analysis, the result (i.e., the temperature distribution) obtained computationally at the microscale model is served as the input in the macroscale model. The main modeling technique used in this simulation is the thermomechanically coupled finite element analysis. In the lower level (microscale), temperature increase caused by the heat flux applied is captured while in the higher level (macroscale) the temperature distribution data is applied to the macroscale model.

In Table. 1, material properties used in the simulations are listed. Simulations for Lamb wave propagation in the plate are performed using a finite element analysis program (FEAP) in a Linux-based workstation with 16 CPU cores.
Fig. 2. Multiscale modeling of hierarchical structure for predicting the thermal wave propagation.

Table 1. Material properties of the aluminum utilized in the simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (Kgm(^{-3}))</td>
<td>2769</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>70.2</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.345</td>
</tr>
<tr>
<td>Conductivity (Wm(^{-1})K(^{-1}))</td>
<td>223.95</td>
</tr>
<tr>
<td>Specific heat (Jkg(^{-1})K(^{-1}))</td>
<td>926.67</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K(^{-1}))</td>
<td>2.31 \times 10^{-5}</td>
</tr>
</tbody>
</table>

**Laser-generated Lamb wave propagation**

In Fig. 3, the out-of-plane displacement in the plate is captured using FEAP in sequential order.

![Lamb waves propagation in the plate](image)

**CONCLUSIONS**

In this paper, the multiscale modeling scheme with the microscale and macroscale models is employed to predict Lamb waves propagation in the isotropic plate. Also, through the analytical model regarding the transient responses of laser pulses, valid comparison between the numerical and analytical methods can be performed. This study serves as a starting point for the more advanced prediction of Lamb waves in the structure. Future work should complement the analytical part of solving the displacement fields.

**REFERENCES**


