Uncertainty Quantification of Riser Fatigue Damage due to VIV using a Distributed Wake Oscillator Model

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Fatigue damage in risers undergoing vortex-induced vibration (VIV) remains a challenging problem → great uncertainty associated with it.

A distributed wake oscillator model (Srinil, 2010) has been proposed and its effectiveness demonstrated for different flow conditions and riser configurations.

Uncertainty in model parameters can influence fatigue damage estimates.

An Uncertainty Quantity (UQ) framework is presented – using up to 3 random variables – to assess fatigue damage uncertainty.

Case studies with different UQ surrogate models are compared vs. Monte Carlo simulation, while addressing efficiency and accuracy.
Distributed wake oscillator model in a top-tensioned riser

1. Cross-flow motion ($v$)

2. Hydrodynamic lift force ($H_y$)

3. Distributed wake oscillator ($Q$)

$Q$: fluid wake variable

Credit: http://www.mcef.ep.usp.br/staff/jmeneg/cesareo/vort2.gif, Cesareo de La Rosa Siqueira, 2005

Uncertainty Quantification

Fatigue damage
Model Formulation

Cross-flow motion, $v$ (Srinil, 2011)

\[ (M + M_a) \frac{d^2 v}{dt^2} + c \frac{dv}{dt} + EI \frac{d^4 v}{dx^4} - T_R \frac{d}{D^2} \frac{d(v)}{dx} \left( 1 - x \frac{DW_e}{TR} \right) \frac{dv}{dx} = H_y(x,t) \]

\[ M: \text{mass of riser} \quad c: \text{viscous damping} \quad EI: \text{flexural rigidity} \quad D: \text{external diameter} \quad W_e: \text{submerged weight} \]

\[ \omega_s: \text{vortex-shedding freq.} = \frac{2 \pi St V}{D} \quad C_L0: \text{lift coefficient of stationary circular cylinder} \]

\[ F: G: \text{empirical wake coefficients} \]

\[ \omega_s: \text{vortex-shedding freq.} = \frac{2 \pi St V}{D} \quad C_L0: \text{lift coefficient of stationary circular cylinder} \]

\[ F \equiv F(\mu, S_G, \gamma, \delta_A, \Psi_A) \]

\[ G \equiv G(F, C_{L0}, S_G, \gamma, \delta_A) \]

\[ \text{Intermediate dimensionless parameters} \]

\[ \Rightarrow \quad F \quad \text{and} \quad G \quad \text{are empirically expressed as functions of system parameters of the structure and the flow.} \]
Riser Dynamics

• Second-order governing PDEs formulated in modal form (under lock-in conditions, $\omega_n \approx \omega_s$).

\[
\begin{align*}
\mathbf{v}(x,t) &= \sum_{n=1}^{\infty} \phi_n(x)f_n(t), & \mathbf{\dot{v}}(x,t) &= \sum_{n=1}^{\infty} \phi_n(x)\dot{f}_n(t) \\
Q_y(x,t) &= \sum_{n=1}^{\infty} \phi_n(x)d_n(t), & \dot{Q}_y(x,t) &= \sum_{n=1}^{\infty} \phi_n(x)\dot{d}_n(t)
\end{align*}
\]

• Standard Galerkin method with zero displacements and curvatures at end boundaries $\Rightarrow f_n(t)$ and $d_n(t)$ obtained using reduced-order models

Assumed based on a Fourier sine-based series (Srinil, 2009)
Marine riser

**Modeling of VIV**

- Stress time series
  - Vortex-induced vibration response by modal analysis
  - Rainflow cycle counting and Miner’s rule
- Fatigue damage at the worst location (QoI)

**Surrogate model for QoI**

- Quantity of interest (QoI)
  - \( X_1 \)
  - \( X_2 \)

**Input random variables**

- Residual of \( A_{\text{max}} : \Delta A_{\text{max}} \)
- Residual of \( \omega_{\text{s,A}} / \omega_n : \Delta \omega^* \)
- Current velocity: \( V \)

**UQ**

- Shifted Generalized Lognormal Distribution (SGLD)
- Fitting of PDF
- Polynomial Chaos Expansion (PCE)
- Polynomial basis function construction
Uncertain Model Parameters, $\Delta A_{max}^*$ and $\Delta \omega^*$

- Based on 64 datasets for $A_{max}$ and 43 datasets for $\omega_{s,A}/\omega_n$, Skop and Balasubramanian (1997) proposed the equations following (fitted curves in the figures):

$$A_{max} = \frac{0.385}{\sqrt{0.12 + S_G^2}},$$

$$\frac{\omega_{s,A}}{\omega_n} = 1.216 + \frac{0.084}{1 + 2.66S_G^2}. $$

- To account for experimental variability, Low and Srinil (2016) defined:

$$A_{max} = A_{max}^* + \Delta A_{max}^*, \quad \text{design value} \rightarrow \text{residual} \rightarrow \text{RV}$$

$$\frac{\omega_{s,A}}{\omega_n} = \omega^* + \Delta \omega^*, \quad \text{design value} \rightarrow \text{residual} \rightarrow \text{RV}$$

Figures from Low and Srinil (2016)
Shifted Generalized Lognormal Distribution (Low, 2013)

- # of matching moments for SGLD: 4 (mean, standard deviation, skewness, and kurtosis)
- Chi-square statistics (1% significance) for SGLD: $\Delta A_{max}^*$: 3.97 (< 15.09) and $\Delta \omega^*$: 6.68 (< 15.09); smallest
Uncertain model parameters: $\Delta A_{max}^*$ and $\Delta \omega^*$

- Both parameters are estimated from datasets → they are connected via the Skop-Griffin parameter, $S_G$
- Correlation coefficient between the two, i.e., $\rho_{\Delta A_{max}^*, \Delta \omega^*} = 0.3607$

Figures from Low and Srinil (2016)
Uncertain $V$ (current velocity)

- Lock-in condition, which happens when the wake and the riser oscillate concurrently, results in large-amplitude oscillations critical in fatigue life prediction of a riser.

\[ \omega_n \approx \omega_s \]

- Thus, current velocity, $V$, is an important variable that is associated with the lock-in condition during vortex shedding and with the vortex shedding frequency.

\[ \omega_s = 2\pi StV/D \]

- According to DNV-RP-F204 (2010), a 0.05-0.10 CoV is recommended.

$\rightarrow V \sim \text{Lognormal (median= 0.4 m/s, CoV = 8%)}$
(Low and Srinil, 2016).
Quantity of Interest (QoI)

- QoI: Accumulated fatigue damage \( D \) at the middle of the riser due to stress (sum of bending and axial contributions)

- \( X \) is a vector of random variables in the estimation of fatigue damage.

\[
QoI: \quad D(X) \quad X \equiv \{ X_1, X_2, X_3 \}, \text{where } X_1 = \Delta A_{max}, X_2 = \Delta \omega^*, X_3 = V
\]

- For convenience, \( D \) will be normalized by its design value:

\[
\bar{D}(X) = \frac{D(X)}{D(X_{design})} \quad \rightarrow \quad X_{design} \quad \begin{cases}
\Delta A_{max}^* = 0 \\
\Delta \omega^* = 0 \\
V = 0.4 \text{ (median value)}
\end{cases}
\]
Fatigue Damage Estimation (Monte Carlo)

Bivariate case (Rosenblatt)

\[ X_1 = F_{X_1}^{-1}[\Phi(\sqrt{1 - \rho_{Y_1Y_2}^2 Q_1 + \rho_{Y_1Y_2} Q_2})] \]
\[ X_2 = F_{X_2}^{-1}[\Phi(Q_2)] \]

\( X \): physical, \( Y \): correlated standard normal, 
\( Q \): uncorrelated standard normal.

Rainflow cycle counting

Post-processing after the simulations

Miner’s rule

pdf of \( D(X) \) by MCS

S-N curve parameters depend on material

\[ D = \frac{1}{a} \sum_{i=1}^{n} S_i^m c_i \]

number of cycles at \( t \)th stress range

\[ D(X) \]

\[ D(X_{\text{design}}) \]
# Fatigue Damage Estimation

1. Cross-flow motion, $\nu$
2. Distributed wake oscillator, $Q_y$

\[ \nu(x, t) \text{ involving a number of system parameters} \]
\[ \text{(including deterministic and uncertain ones)} \]

- solve coupled PDEs with uncertain RVs $(\Delta A^*_{max}, \Delta \omega^*, V)$

- rainfall-cycle counting; Miner’s rule

\[ \bar{D} \text{ (fatigue damage) implicitly related to RVs} \]

→ MCS and UQ
Polynomial Chaos Expansion (PCE)

• For UQ, uncertain QoI is expressed, using PCE:

\[ \bar{D}(X) = \sum_{i=0}^{\infty} c_i \Phi_i(\Psi(X)) \]

\( \Phi_i \) is the \( i^{th} \) multivariate orthogonal polynomial function

functions of input random variables

coefficients to be estimated

• A truncated PCE for \( \bar{D} \) that involves polynomials up to order \( p \) is:

\[ \bar{D}(X) \approx \bar{D}^{PCE}(X) = \sum_{i=0}^{N-1} c_i \Phi_i(\Psi(X)), \quad N = \sum_{k=0}^{p} \binom{N_X + k - 1}{k} \]

\( N \): number of unknown PCE coefficients in polynomials of order not exceeding \( p \)

\( N_X \): number of physical variables treated as random

• In the study, multivariate Hermite polynomials are basis functions (Askey scheme)

• The \( i^{th} \) multi-dimensional Hermite polynomial is:

\[ \overline{He}_i(Q_j) = (-1)^i \frac{1}{\varphi(Q_i)} \frac{d^i \varphi(Q_j)}{dq^i} \]

\[ \varphi(Q_j) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Q_j^2}{2}} \]

\[ He_i(Q) = \prod_{j=1}^{N_X} He_{\alpha_i}(Q_j) \]

standard normal variable

multi-index
Polynomial Chaos Expansion (PCE)

- Spectral projection (quadrature-based integration) is used to compute the coefficients, $c_i$:

$$c_i = \frac{E[D(X)He_i(Q)]}{E[He_i^2(Q)]}$$

- The denominator may be derived analytically; the numerator is approximated as

$$E[D(X)He_i(Q)] = \int \cdots \int D(X)He_i(Q) f_X(x) dx_1 dx_2 \cdots dx_{N_X}$$

approximation by Gauss-Hermite quadrature rule

$$\sum_{x_{1k}=1}^{n_q} \sum_{x_{2k}=1}^{n_q} \cdots \sum_{x_{(N_X)k}=1}^{n_q} f(x_{1k}, x_{2k}, \cdots, x_{(N_X)k}) He_i(Q(x_{1k}, x_{2k}, \cdots, x_{(N_X)k})) w_{1k} w_{2k} \cdots w_{(N_X)k}$$

quadrature points

weights
PCE Example: Building Surrogate Model for QoI

- 2 variables, order 2, 5 quadrature points, Hermite polynomials

map to standard normal space

\( X = \{X_1, X_2\}, \ p = 2, \ N_q = 5 \)

\( Q = \{Q_1, Q_2\} \)

\[
\bar{D}(X) \approx \bar{D}^{PCE}(X) = \sum_{i=0}^{N-1} c_i \Phi_i(\Psi(X)) = \sum_{i=0}^{5} c_i He_i(Q_1, Q_2) = c_0 He_0 + c_1 He_1 + c_2 He_2 + c_3 He_3 + c_4 He_4 + c_5 He_5
\]

\[
N = \sum_{k=0}^{2} \binom{N_x + k - 1}{k} = \frac{2 + k - 1}{k} = 6
\]

Univariate polynomials

\[
\varphi(Q_j) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Q_j^2}{2}}
\]

\[
He_i(Q_j) = (-1)^i \frac{1}{\varphi(Q_j)} \frac{d^i \varphi(Q_j)}{dq^i}
\]

\[
He_0(Q_j) = 1
\]

\[
He_1(Q_j) = Q_j
\]

\[
He_2(Q_j) = Q_j^2 - 1
\]

Bivariate polynomials

<table>
<thead>
<tr>
<th>i</th>
<th>multi-index</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
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<tr>
<td>0</td>
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<td>0</td>
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<td>1</td>
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</tr>
<tr>
<td>5</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
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</table>

\[
He_0(Q_1, Q_2) = He_0(Q_1) \cdot He_0(Q_2) = 1
\]

\[
He_1(Q_1, Q_2) = He_1(Q_1) \cdot He_0(Q_2) = Q_1
\]

\[
He_2(Q_1, Q_2) = He_0(Q_1) \cdot He_1(Q_2) = Q_2
\]

\[
He_3(Q_1, Q_2) = He_2(Q_1) \cdot He_0(Q_2) = Q_1^2 - 1
\]

\[
He_4(Q_1, Q_2) = He_1(Q_1) \cdot He_1(Q_2) = Q_1 \cdot Q_2
\]

\[
He_5(Q_1, Q_2) = He_0(Q_1) \cdot He_2(Q_2) = Q_2^2 - 1
\]
PCE Example: Building Surrogate Model for QoI

\[
\bar{D}(X) \approx \bar{D}^{PCE}(X) = c_0 H e_0 + c_1 H e_1 + c_2 H e_2 + c_3 H e_3 + c_4 H e_4 + c_5 H e_5
\]

Multiply by \( H e_i \)

\[
E[\bar{D}(X) \cdot H e_i] = E[(c_0 H e_0 + c_1 H e_1 + c_2 H e_2 + c_3 H e_3 + c_4 H e_4 + c_5 H e_5) \cdot H e_i]
\]

Estimate \( c_i \)

\[
c_i = \frac{E[\bar{D}(X(Q))H e_i(Q_1, Q_2)]}{E[H e_i^2(Q_1, Q_2)]}
\]

\( 5^2 \) quadrature

<table>
<thead>
<tr>
<th>( c_0 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( c_5 )</th>
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</thead>
<tbody>
<tr>
<td>1.223</td>
<td>0.406</td>
<td>-0.577</td>
<td>0.0896</td>
<td>-0.183</td>
<td>0.0952</td>
</tr>
</tbody>
</table>

• Resulting 2nd-order PCE model involving 2 uncertain parameters is:

\[
\bar{D}(X_1, X_2) \approx \bar{D}^{PCE}(Q_1(X_1), Q_2(X_2)) = 1.223 + 0.406 Q_1 - 0.577 Q_2 + 0.0896 (Q_1^2 - 1) - 0.183 Q_1 Q_2 + 0.0952 (Q_2^2 - 1).
\]
Comparisons of $\bar{D}$ at Gauss-Hermite quadrature points: $Q_{qp} = \{-2.857, -1.356, 0, 1.356, 2.857\}$

For the true $\bar{D}$, a mapping from standard normal ($Q$) to physical space (fitted to SGLD) is performed.

The blue and red dots in the graphs represent $\bar{D}$ at the quadrature points, which show that a PCE order-2 model with 25 quadrature points yields accurate fatigue damage prediction.
Fatigue Damage Estimation using PCE

- Surrogate model for normalized fatigue damage constructed in terms of uncertain model parameters
Illustration (Key Riser and Wake Data)

- variables: deterministic, uncertain
- types: structure, fluid, combined

\[ M: \text{mass of riser} = 87.73 \text{ kg/m} \]
\[ M_a: \text{added mass of fluid} = 57.82 \text{ kg/m} \]
\[ EI: \text{flexural rigidity} = 1.85 \times 10^7 \text{Nm}^2 \]
\[ W_e: \text{submerged weight} = 146 \text{ kN} \]
\[ D: \text{external diameter} = 0.268 \text{ m} \]
\[ T_R: \text{top tension} = 1.21 \text{ MN} \]
\[ L: \text{length of riser} = 500 \text{ m} \]
\[ \xi: \text{damping ratio} = 0.01 \]

\[ C_{L0}: \text{lift coefficient} = 0.28 \]
\[ \rho: \text{fluid density} = 1025 \text{ kg/m}^3 \]
\[ V: \text{uniform current velocity} = 0.4 \text{ (median)} \]
\[ St: \text{Strouhal number} = 0.2 \]

\[ \omega_s: \text{vortex-shedding freq.} = 1.88 \text{ rad/s} \]
\[ S_G: \text{Skop-Griffin parameter} = 0.0624 \]
\[ \bar{\mu}: \text{mass ratio} = \frac{\rho D^2}{8\pi^2 St^2 (M + M_a)} = 0.1602 \]
Case Study (1 variable): Q-Q plots

computationally inexpensive for propagating uncertainty in PCE; efficient for investigating model parameters

- In the Q-Q plots for each parameter, PCE quantiles with all orders (1, 2, 5, and 10) show good agreement with MCS in the body (middle); however, in the tails, there are noticeable differences

- The higher order PCE cases (5 and 10) are in good agreement with MCS while low-order PCE models (1 and 2) cannot capture stochastic characteristics of the QoI

<table>
<thead>
<tr>
<th>N: # of quadrature pts</th>
<th># of samples for MCS: 2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># of samples for PCE: 100,000</td>
</tr>
</tbody>
</table>
Case Study (1 variable): Exceedance probability plots

• PCE models of all orders (1, 2, 5, and 10) for each parameter provide fairly accurate probability estimates compared with MCS for moderate damage situations ($d \sim 0 - 1.5$).

• For $d > 1.5$, deviations from MCS are greater but it should be noted that because of sample size, the MCS estimates also have greater uncertainty at these $d$ levels.

• For second uncertain variable, $\Delta \omega^*$, the PCE models with order 5 and 10 show better agreement with MCS, than first uncertain variable, $\Delta A_{max}^*$. 
To assess the relative importance between the two variables, comparisons between MCS with one variable and with two variables are performed.

\( \Delta A_{max}^* \) is less influential than \( \Delta \omega^* \) because, when \( \Delta A_{max}^* \) is considered not uncertain (fixed at its median value), there is smaller deviation relative to MCS with two variables.
For all cases (N= 25, 64, and 100), damage PDFs for PCE with polynomial order 5 and 10 show good agreement with MCS whereas those with polynomial orders 1 and 2 are clearly not satisfactory.

In the exceedance plots, the PCE model with polynomial order 5 and 10 again show good agreement with MCS; PCE exceedance probability curves for those cases lie within the MCS 99% confidence intervals.

Increasing polynomial order is not always beneficial due to overfitting.
Higher $V$ results in greater fatigue damage.

Due to the positive correlation between $\Delta A_{max}^*$ and $\Delta \omega^*$, $\overline{D}$ is lower than for the case with zero correlation; this because $\overline{D}$ increases with $\Delta A_{max}^*$, but decrease with $\Delta \omega^*$.
• PCE model with polynomial order 5 shows very good agreement with MCS over all damage ranges.

• Overfitting with order-10 PCE scheme results in inferior performance, particularly at low damage levels, relative to order-5 scheme.

• Exceedance probability plots show that the PCE models with polynomial orders 2, 5, and 10 match MCS predictions quite well. Over the range, \(0 < d < 6\), all the PCE models except the order-1 scheme predict exceedance probabilities that lie within MCS 99% confidence bounds.
Summary of UQ Surrogate Models

MCS is performed using 3 random variables $(\Delta A_{\text{max}}^*, \Delta \omega^*, V)$

All PCE models are order-5 and use $5^{N_X}$ quadrature points ($N_X = \# \text{ of variables treated as random}$)

- Different PCE curves suggest contrasting levels of influence of the variables
- $V$ influence is greater for $d > 1$ (comparing green and red lines)
- An order-5 PCE scheme with $5^3$ quadrature points appear to be accurate; damage probability levels are within 99% confidence intervals of MCS
Summary of UQ Surrogate Models (Computational Effort)

<table>
<thead>
<tr>
<th>Variables</th>
<th>MCS</th>
<th>PCE</th>
<th>MCS:PCE EFFORT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta A^{*}_{\text{max}}$</td>
<td>2,000</td>
<td>5</td>
<td>400:1</td>
</tr>
<tr>
<td>$\Delta \omega^{*}$</td>
<td>2,000</td>
<td>5</td>
<td>400:1</td>
</tr>
<tr>
<td>$\Delta A^{<em>}_{\text{max}}$ and $\Delta \omega^{</em>}$</td>
<td>20,000</td>
<td>64</td>
<td>312.5:1</td>
</tr>
<tr>
<td>$\Delta A^{<em>}_{\text{max}}$, $\Delta \omega^{</em>}$, and $V$</td>
<td>20,000</td>
<td>125</td>
<td>160:1</td>
</tr>
</tbody>
</table>

- PCE is computationally much less expensive than MCS to predict fatigue damage.

- Also, for accurate uncertainty propagation, PCE is more efficient than MCS and involves significantly fewer time-consuming VIV response simulations.
Conclusions

- Fatigue damage estimation due to VIV in a marine riser is carried out by employing Polynomial Chaos Expansion with various choices for model parameters that are treated as uncertain.

- A distributed wake oscillator model is used to perform numerical studies with the selected uncertain parameters.

- When two key model parameters – i.e., $\Delta A_{max}^*, \Delta \omega^*$ – are considered as uncertain, a PCE order-5 model with $5^2 = 25$ quadrature points is found to be accurate when compared against MCS predictions.

- When current velocity, $V$, is added as a third random variable, again a PCE order-5 scheme is found to be accurate with $5^3 = 125$ quadrature points.
Tusen takk