Thermomechanical Analysis of Laser-generated Ultrasonics in Thin Structures by Finite Element Method

Master Thesis Defense, Department of Civil and Environmental Engineering, KAIST, Daejeon, Korea

Wednesday, December 18, 2013 4:00 PM
KAIST W16 Geo-centrifuge 2F Seminar Room

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Thesis Structure

1. Introduction
2. Theories
3. Numerical Model
4. Analytical Model
5. Applications
6. Conclusions
Thesis Structure

1. Introduction
   The objectives and motivations of the thesis

2. Theories

3. Numerical Model

4. Analytical Model

5. Applications

6. Conclusions
Thesis Structure

1. Introduction
2. Theories 
   Basis for understanding the concepts in the thesis
3. Numerical Model
4. Analytical Model
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1. Introduction
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3. Numerical Model
   The numerical model for predicting Lamb waves
4. Analytical Model
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1. Introduction
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   The analytical model of transient Lamb waves
5. Applications
6. Conclusions
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1. Introduction

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5. Applications
   - Investigation of the effect of the presence of damage in a pipe

6. Conclusions
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Summary and conclusions
Thesis Structure

1. Introduction

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Motivations

Longitudinal crack

Seongsu Bridge collapse in Korea, 1994
I-35W Mississippi River-bridge collapse in USA, 2007
A giant oil spill in the Gulf of Mexico, 2010

Fukushima accident, 2011
Hydrofluoric acid gas leak in Korea, 2012

In the wake of severe disasters, many civil engineers have intensified their efforts to develop effective techniques for detecting damages in structure.
Structural Health Monitoring

Among Techniques of NDT and SHM

- Ultrasonic wave
  Suitable for testing of thin structures

- Using piezoelectric transducer
  Tone-burst signal

- Laser-generated ultrasound
  Broadband signal

Aging Infrastructure

Damage Detection

Image: National Association of Water Companies

Structural health monitoring system can detect and sound an alarm if there is any danger to the safe operations of infrastructure.
Laser Ultrasonic Techniques

Among the techniques of nondestructive testing, ultrasound based method is especially suitable for a testing of thin structures such as plates and pipes.
Motivations

- Number of research papers about laser-generated Lamb waves

Small numbers of studies are found that are closely related to laser-generated Lamb waves in analytical or numerical way.
Thesis objectives

1. To develop schemes and methods for numerical simulations of Lamb waves propagation in thin structures.

2. To show the analytic solution of transient response of laser-generated Lamb waves.

3. To develop the criteria for the evaluation of the damage in structures.
Infrastructure → Thin structures

Complexity of Infrastructure

Basic Components: Plates and Pipes

Ultrasound based method is especially suitable for a testing of thin structures because guide waves in the through-thickness direction are sensitive to small defects in the structures.
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**Dynamic elastic waves**

- Incidence of a pulse laser

1. Quasi-static thermo-elastic surface deformation is induced.
2. Dynamic elastic waves are excited.
Ultrasound generation mechanism

- Incidence of a pulse laser

Thermal diffusivity or Optical penetration

Thermo-elastic subsurface source

Solid

Longitudinal waves, shear waves, Rayleigh and Lamb wave are generated.

- Navier’s equation (Displacement equations of motion)

\[(\lambda + \mu)\nabla (\nabla \cdot u) + \mu \nabla^2 u = \rho \ddot{u}\]

In most of the works about solving ultrasound analytically, thermo-elastic equation is solved by applying double integral transforms or the Green’s function formalism.
Finite element method

The finite element method, a proven technique for dealing with a diverse range of engineering problems, is an immensely powerful numerical technique\(^1\).

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Thermo-mechanically coupled finite element formulation

- **Governing equation**
  \[
  \rho \ddot{u}_i - \frac{\partial}{\partial x_j} \sigma_{ij} = f_i
  \]
  Equation of motion in a local form
  \( \rho \) : mass density
  \( f_i \) : body force

- **Constitutive equation**
  \[
  \sigma_{ij} = C_{ijkl} \varepsilon_{kl} - \beta_{ij} T
  \]
  Thermo-mechanically coupled constitutive equations
  \( C_{ijkl} \) : elasticity tensor

- **Compatibility equation**
  \[
  \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
  \]
  Strain-displacement relationship
  \( \varepsilon_{ij} \) : strain tensor

- **Entropy equation**
  \[
  T_0 \dot{s} = -\frac{\partial q_i}{\partial x_i}
  \]
  Local balance equation of entropy
  \( T_0 \) : equilibrium temperature

- **Heat conduction equation**
  \[
  q_i = -k_{ij} \frac{\partial T}{\partial x_j}
  \]
  Fourier’s heat conduction law
  \( k_{ij} \) : thermal conductivities

---

Thermo-mechanically coupled finite element formulation

Virtual work principle, integration by parts and divergence theorem

\[ \int_V \varepsilon_{ij} \sigma_{ijkl} \varepsilon_{kl} dV - \int_V \varepsilon_{ij} \beta_{ij} T dV + \int_V \delta u_i \rho \ddot{u}_i dV = \int_{\Gamma} \sigma_{ij} n_j \delta u_i dA + \int_V f_i \delta u_i dV \]

\[ T_0 \int_V \beta_{ij} \dot{\varepsilon}_{ij} T dV + \int_V c E \dot{\theta} \delta T dV + \int_V \delta \theta_i k_{ij} T_{,j} dV = -\int_{\Gamma} q_i n_i \delta T dA \]

Discretization for the finite element formulation

\[ u = H_u U \quad \theta = H_\theta \Theta \quad \varepsilon = H_u U \quad \theta_i = H_\theta \Theta \]

Finite element equations

\[ M \ddot{U} + K_{uv} Q + K_{uu} U = F \]

\[ C_{q\varepsilon} \dot{U} + C_{qq} \dot{Q} + K_{qq} Q = Q \]

u: displacement vector
q: temperature
e: strain vector
U: nodal displacement vector
Q: nodal temperature vector
H_u: displacement interpolation matrix
H_q: temperature interpolation matrix
B_u: strain-displacement matrix
B_q: temperature-gradient interpolation matrix
F: nodal force vector

**Bottom-up Analysis for a Multi-scale Modeling**

In order to obtain the comprehensive understanding, one should have an insight on how ultrasonic waves are formed and developed in the **thermo-mechanical point of view**.
Schematic of circular crested waves

1. Shear traction

2. Normal traction

Lamb waves are analyzed by studying the transient response of a plate subjected to a distributed normal force.

Integral Transform Method (ITM)\(^1\)

- Navier's equation
- Ordinary differential equation
  - Integral transformation
  - Analytical solution
  - Transformed response
  - Inverse integral transformation
  - Response
  - Finite Element Analysis

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We need a new methodology

1. Thermal wave propagation induced by nanosecond laser pulses in structures is very difficult to calculate since the duration of the pulse is extremely short.

2. Therefore, the temperature response of the material is underestimated as a very small value.

3. To model the heat flux for few nanoseconds, the numerical model should be refined with extremely small elements, and it requires enormous amount of time.
Multi-scale modeling

Hierarchical description of the laser pulse excitation at different scales

1. To alleviate numerical cost, a bottom-up analysis for a multi-scale modeling is proposed to accurately predict thermo-mechanical wave propagation.

2. Two different scale models (the micro-scale and the macro-scale models) are introduced.
Numerical model

- Multi-scale thermal analysis

Small-scale model

Plate

Pipe

Large-scale global model
**Multi-scale thermal analysis (Micro scale)**

- **Simulation conditions**
  - Young’s modulus: 70.2 GPa
  - Density: 2769 Kgm\(^{-3}\)
  - Poisson’s ratio: 0.345

![Temperature profile graph]

- Temperature profile
  - Time (μs): 0 to 10
  - Temperature (°C): -100 to 500

![Diagram showing microscale thermal analysis]
Element segmentation for computation

- Monolithic scheme vs Partitioning scheme

\[
\begin{align*}
M\ddot{\mathbf{U}} + K_{\text{M}} \mathbf{Q} + K_{\text{Um}} \mathbf{U} &= \mathbf{F} \\
C_{\text{M}} \dot{\mathbf{U}} + C_{\text{Q}} \dot{\mathbf{Q}} + K_{\text{Q}} \mathbf{Q} &= \mathbf{Q}
\end{align*}
\]

Discretize the entire domain into a thermo-mechanical region and a mechanical region\(^1\).

The implicit time integration scheme is employed in the thermo-mechanical domain while the explicit time integration scheme is used in the mechanical region\(^1\).

Element segmentation for computation

- Partitioning scheme

Partition of the pipe into a thermo-mechanical region and a mechanical region

Same procedure for the element segmentation is applied to the pipe structure.
**Numerical condition (geometry)**

- **Pipe**

```
| 350 mm | 300 mm | 350 mm |
```

Geometry of the pipe without a notch
Numerical condition (geometry)

- Plate

Geometric property of an aluminum plate for the simulation
Lamb wave propagation in a pipe

8.00 ns | Laser excitation

12.20 μs | Interaction in the thermo-mechanical domain

28.80 μs | Interaction in the mechanical domain

80.60 μs | Reflection on the boundaries
Lamb wave propagation in a plate

5.00 ns | Laser excitation

12.40 μs | Interaction in the thermo-mechanical domain

26.30 μs | Interaction in the mechanical domain

60.90 μs | Reflection on the boundaries
Numerical model

Simulation validation

Comparison between the simulation result and the experimental result\(^1\) (pipe). They are showing good agreement.

Simulation validation

Comparison between the experimental result\textsuperscript{1} and the simulation result (plate). They are showing good agreement.

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**Theoretical model**

- **Equations of motion**

\[
(\lambda + 2\mu) \frac{\partial \Delta}{\partial r} + \mu \frac{\partial \Omega}{\partial z} = \rho \frac{\partial^2 u_r}{\partial t^2}
\]

\[
(\lambda + 2\mu) \frac{\partial \Delta}{\partial z} - \frac{\mu}{r} \frac{\partial (r\Omega)}{\partial r} = \rho \frac{\partial^2 u_z}{\partial t^2}
\]

\[\lambda: \text{1st Lame parameter}\]

\[\mu: \text{2nd Lame parameter}\]

\[\Delta: \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \quad \text{Dilatation}\]

\[\Omega: \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \quad \text{Rotation}\]

- **Transformed displacement** (our interest is to obtain the transformed displacement induced by the external force \(f(r,t)\).)

\[
\hat{u}_z(z,k,\omega) = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u_z(z,r,t)rJ_0(kr)e^{j\omega t} \, dr \, dt
\]

\[
\hat{u}_r(z,k,\omega) = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u_r(z,r,t)rJ_1(kr)e^{j\omega t} \, dr \, dt
\]

\[J_0(kr): \text{Bessel functions of the zeroth order}\]

\[J_1(kr): \text{Bessel functions of the first order}\]

---

Theoretical model

- Fourier-Hankel transformation pair

\[
\hat{u}_z(z, k, \omega) = \int_{-\infty}^{+\infty} e^{j\omega t} \, dt \int_{0}^{+\infty} u_z(z, r, t) r J_0(kr) \, dr
\]

\[
\hat{u}_r(z, k, \omega) = \int_{-\infty}^{+\infty} e^{j\omega t} \, dt \int_{0}^{+\infty} u_r(z, r, t) r J_1(kr) \, dr
\]

- Inverse Fourier-Hankel transformation pair

\[
u_z(z, k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-j\omega t} \, dt \int_{0}^{+\infty} \hat{u}_z(z, k, \omega) k J_0(kr) \, dk \, \, d\omega
\]

\[
u_r(z, k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-j\omega t} \, dt \int_{0}^{+\infty} \hat{u}_r(z, k, \omega) k J_1(kr) \, dk \, \, d\omega
\]

\[J_0(kr) : \text{Bessel functions of the zeroth order}\]

\[J_1(kr) : \text{Bessel functions of the first order}\]

Theoretical model

- Boundary conditions

\[
\sigma_{zz}(r,t) = \begin{cases} 
  f(r,t) & \text{at } z = +h \\
  0 & \text{at } z = -h 
\end{cases}
\]

\[
\sigma_{rz}(r,t) = 0 \quad \text{at } z = \pm h
\]

- Excitation function

\[
[H(r) - H(r - d/2)] \delta(t)
\]

\(H(x) = \text{Heaviside step function}\)

\(d = \text{beam diameter}\)
Theoretical model

- Transformed solution

\[
\hat{u}_r(z, k, \omega) = \{A_s k \cosh(\alpha z) - D_s \beta \cosh(\beta z)\} + \{B_a k \sinh(\alpha z) - C_a \beta \sinh(\beta z)\}
\]

\[
\hat{u}_z(z, k, \omega) = -\{A_s \alpha \sinh(\alpha z) - D_s k \sinh(\beta z)\} - \{B_a \alpha \cosh(\alpha z) - C_a k \cosh(\beta z)\}
\]

- Inverse Fourier-Hankel transform using the residue theorem

\[
u_z(h, x, t) = \int_{-\infty}^{+\infty} \sum_{k_s} H_z^s(h, \omega) \hat{f}(k, \omega) k J_0(kr)e^{-j\omega t} d\omega + \int_{-\infty}^{+\infty} \sum_{k_a} H_z^a(h, \omega) \hat{f}(k, \omega) k J_0(kr)e^{-j\omega t} d\omega
\]

\[
u_r(h, x, t) = \int_{-\infty}^{+\infty} \sum_{k_s} H_r^s(h, \omega) \hat{f}(k, \omega) k J_1(kr)e^{-j\omega t} d\omega + \int_{-\infty}^{+\infty} \sum_{k_a} H_r^a(h, \omega) \hat{f}(k, \omega) k J_1(kr)e^{-j\omega t} d\omega
\]

Calculation of individual wave modes.

There exist a number of wave modes.

1. Draw the dispersion curve for each wave mode.
2. Obtain relation between angular frequencies and wavenumbers.
3. Carry out the integration in the analytic solution.
Antisymmetric mode
Symmetric model
Superposition of each mode

\[ A + S \]
Superposition of each mode

Experimental Result

Numerical Result 1
Multi-scale Modeling (Thermal loading)

Numerical Result 2
General FEM Modeling (Pressure loading)

Analytical Result
Comparison 1

The multi-scale finite element modeling (thermal loading) shows a good agreement with the experimental result.
The general finite element modeling (pressure loading) shows a good agreement with the analytic result.
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Modeling of a Pipe with a Notch

- Pipe with a longitudinal notch

- Pipe with a circumferential notch
Damage scenario

- Different notches

As the depth of the notch increased by 1mm to 4mm in each successive case from case 1 to case 4, the amount of energy distracted by a notch when waves propagate from a source also increases.
Applications

Investigation of the energy dissipation

Cases with a longitudinal notch

\[ \Omega = \frac{A_S}{A_T} \]

- the total area of the frequency range.
- the area of the particular frequency range.

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<tr>
<th>Case number</th>
<th>RMSD (x10^{-11})</th>
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<tr>
<td>1</td>
<td>2.1525</td>
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<tr>
<td>2</td>
<td>2.3400</td>
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<tr>
<td>3</td>
<td>3.4911</td>
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<tr>
<td>4</td>
<td>5.1041</td>
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</table>

Investigation of the energy dissipation

◆ Cases with a circumferential notch

\[ \Omega = \frac{A_S}{A_T} \]

\( A_S \): the total area of the frequency range.
\( A_T \): the area of the particular frequency range.

0.42 MHz to 0.64 MHz

<table>
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<tr>
<th>Case number</th>
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<td>3</td>
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<table>
<thead>
<tr>
<th>Case number</th>
<th>( \Omega^1 )</th>
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<tbody>
<tr>
<td>Intact</td>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>0.1886</td>
</tr>
<tr>
<td>3</td>
<td>0.2251</td>
</tr>
<tr>
<td>4</td>
<td>0.2015</td>
</tr>
</tbody>
</table>

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Conclusions

1. A numerical methodology based on **multi-scale modeling** is presented.

2. The proposed simulation procedures are verified by **comparing experimental and analytical results**.

3. A methodology for **damage detection** for pipe-like structures is provided.
Recommendations for Further Work

1. Applying the multi-scale modeling scheme into a curved pipe.
3. Damage detection in case of multi notches.
Acknowledgments

◆ Korea Agency for Infrastructure Technology Advancement (KAIA)

◆ Korea ministry of land, transport and maritime affairs (MLTM)

  Mr. Hyeong Uk Lim has been supported by U-City master and doctor course grant program.

◆ Partially supported by KAIST EEWS program

◆ Supported by Korea Institute of Energy Technology Evaluation and Planning (KETEP)
End of the Presentation

Thank You for Your Kind Attention

Hyeong Uk Lim

Master student
Back up slides
Multi-scale thermal analysis (Micro scale)

- Simulation conditions
  
  - Young’s modulus: 70.2 GPa
  - Density: 2769 Kgm⁻³
  - Poisson’s ratio: 0.345
  - Thermal expansion coefficient: 2.31x10⁻⁵ K⁻¹
  - Time step: 5x10⁻¹⁰ for the first 40 steps, 1x10⁻⁸ for the rest 50000 steps
Theoretical model

- Transformed solution
  \[
  \hat{u}_r(z, k, \omega) = \left\{ A_s k \cosh(\alpha z) - D_s \beta \cosh(\beta z) \right\} + \left\{ B_a k \sinh(\alpha z) - C_a \beta \sinh(\beta z) \right\}
  \]
  \[
  \hat{u}_z(z, k, \omega) = -\left\{ A_s \alpha \sinh(\alpha z) - D_s k \sinh(\beta z) \right\} - \left\{ B_a \alpha \cosh(\alpha z) - C_a k \cosh(\beta z) \right\}
  \]

- Inverse Fourier-Hankel transform using the residue theorem
  \[
  u_z(h, x, t) = \int_{-\infty}^{+\infty} \sum_{k_s} H_s^z(h, \omega) \hat{f}(k, \omega) k J_0(k r) e^{-j\omega t} d\omega + \int_{-\infty}^{+\infty} \sum_{k_a} H_a^z(h, \omega) \hat{f}(k, \omega) k J_0(k r) e^{-j\omega t} d\omega
  \]
  \[
  u_r(h, x, t) = \int_{-\infty}^{+\infty} \sum_{k_s} H_s^r(h, \omega) \hat{f}(k, \omega) k J_1(k r) e^{-j\omega t} d\omega + \int_{-\infty}^{+\infty} \sum_{k_a} H_a^r(h, \omega) \hat{f}(k, \omega) k J_1(k r) e^{-j\omega t} d\omega
  \]

\[
\left\{
\begin{align*}
H_s^r(h, \omega) &= \frac{j(k_s^4 - \beta^4) \cosh(\alpha h) \sinh(\beta h)}{8k_s \mu \Delta_s'} \\
H_z^s(h, \omega) &= \frac{-j\alpha(k_s^2 - \beta^2) \sinh(\alpha h) \sinh(\beta h)}{4 \mu \Delta_s'}
\end{align*}
\right.
\]

\[
\left\{
\begin{align*}
H_s^a(h, \omega) &= \frac{j(k_s^4 - \beta^4) \sinh(\alpha h) \cosh(\beta h)}{8k_s \mu \Delta_a'} \\
H_z^a(h, \omega) &= \frac{-j\alpha(k_s^2 - \beta^2) \cosh(\alpha h) \cosh(\beta h)}{4 \mu \Delta_a'}
\end{align*}
\right.
\]

Material responses