Numerical Simulation of Guided Waves in Pipe-like Structures by Noncontact Laser Pulse

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ABSTRACT

This paper studies the generation of ultrasonic waves in pipe-like structures using numerical simulation. Thermo-elastic guided waves are generated by using a laser pulse and nondestructive evaluation of structures is performed. Originally, it takes much time in analyzing guided waves in pipe-like structures using the existing numerical methods in addition to the difficulties in solving analytically. For those reasons, the methodology, separating the entire domain into two regions, so called thermo-mechanical and mechanical sub-regions, is introduced to improve the efficiency of the numerical calculation. The calculated results are compared with experimental observation.

INTRODUCTION

Almost structures such as the building, bridge and plant are much built during the industrial-age, the 1960s, in Korea. Since specialists on structure maintenance estimate concrete structure’s life-cycle is 60 years and steel’s life-cycle is 50 years, those structures were superannuated. Therefore, various methods are used in industry to diagnose the life span and integrity of structures. Also, many researchers make a great effort to develop new methods of integrity diagnosis of structures.

Researchers make a full use of methods of using Lamb waves to estimate integrity of structures. Lee et al. proposed method for rockbolt integrity inside tunnel using Fourier and wavelet transform. Hosseini and Ulrich presented a method for numerical simulation of Lamb wave propagation in honeycomb sandwich panels. Lamb waves have also been employed for various defects such as holes and damages in carbon fiber composite. Lamb waves are researched to estimate integrity of structures above.

Apart from the previously mentioned, Lamb waves can be utilized as a source of non-contact NDT (Non-destructive testing) techniques. To overcome the limitations of the existing non-destructive inspection,
many researchers use guided waves by laser pulse. When using guided waves induced by laser pulse, we do not have to contact structures to monitor integrity in pipe-like structures. Lee et al. presented method of monitoring pipe line using optical-fiber guided laser ultrasonic. Like this method, many non-contact technique researches use Lamb wave for the evaluation of rectangular plates, pipes and even thick plates.

In this paper, we describe finite element analysis of guided waves by non-contact laser excitation. Multi-scale method and the methodology separating the entire domain into two regions apply to numerical model to improve the efficiency of the numerical calculation. The goal of these all works is to increase efficiency and accuracy of calculation.

THEORIES

Finite element formulation for thermo-elastic analysis.

The finite element method is used for a wide range of engineering problem. The subject has been studied over the past several decades to analyze wave propagation in the solid media. We have to consider the coupled thermo-elasticity to model Lamb waves excited by laser pulse. First, three equations such as the equilibrium equation, the boundary conditions and the constitutive equation are needed.

The equilibrium equation of motion in local form and the local balance equation of entropy are written, respectively as

\[ \sigma_{ij} + F_i = \rho \dot{e}_{ij} \]  

\[ T_i \dot{S} = -q_i, \]  

where \( \sigma_{ij} \) are stresses, \( F_i \) are body forces, \( \rho \) is the density, and \( u_i \) are displacements. Also, \( T_0 \) is the equilibrium temperature, \( \dot{S} \) is the entropy rate per unit volume, and \( q_i \) is the thermal flux.

For the boundary conditions, both mechanical and thermal boundary conditions must be satisfied. The entire boundary \( \Gamma \) can be decomposed as \( \Gamma = \Gamma_\sigma \cup \Gamma_u = \Gamma_0 \cup \Gamma_\theta \). The null set \( \emptyset \) also can be decomposed as \( \emptyset = \Gamma_\sigma \cap \Gamma_u = \Gamma_0 \cap \Gamma_\theta \). The traction boundary condition, \( \Gamma_\sigma \), and the displacement boundary condition, \( \Gamma_u \), are given as

\[ \sigma_{ij} n_j = \bar{\sigma}_i \text{ on } \Gamma_\sigma, \]  

\[ u_i = \bar{u}_i \text{ on } \Gamma_u, \]  

where \( \bar{\sigma}_i \) are the prescribed surface tractions, and \( \bar{u}_i \) are the prescribed displacements. The heat flux boundary \( \Gamma_\sigma \) and the temperature boundary \( \Gamma_\theta \) are specified as

\[ q_i n_i = \bar{Q} \text{ on } \Gamma_\theta, \]
\[ \theta = \bar{\theta} \text{ on } \Gamma_\theta, \]  
(6)

where \( \bar{Q} \) and \( \bar{\theta} \) are the prescribed heat flux and temperature specified at thermal boundaries.

The constitutive equations are stated as

\[ \sigma_{ij} = C_{ijkl} \epsilon_{kl} - \beta_{ij} \theta, \]  
(7)

\[ s = \beta_{ij} \epsilon_{ij} + \frac{cE}{T_0} \theta, \]  
(8)

where \( C_{ijkl} \) is the elasticity tensor, \( \epsilon_{ij} \) is the strain tensor, \( cE \) is the specific capacity, and \( \beta_{ij} \) is the thermo-mechanical coupling tensor which is of a form for isotropic materials as

\[ \beta = \begin{bmatrix} 3\kappa \alpha & 0 & 0 \\ 0 & 3\kappa \alpha & 0 \\ 0 & 0 & 3\kappa \alpha \end{bmatrix}, \]  
(9)

where \( \kappa \) and \( \alpha \) denote the bulk modulus and the coefficient of thermal expansion, respectively.

The strain-displacement relation is expressed as

\[ \epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \]  
(10)

The Fourier’s heat conduction law is given by

\[ q_{ij} = -k_{ij} \theta_j, \]  
(11)

where \( k_{ij} \) is the thermal conductivity tensor as

\[ k = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}. \]  
(12)

Applying the principle of virtual work and the principle of virtual temperature to Equations (1) and (2), respectively, we have

\[ \int_V (\sigma_{ij,j} + F_i - \rho \ddot{u}_i) \delta u_i dV = 0, \]  
(13)

\[ \int_V (T_{ij,j} - \kappa_{ij} \theta_j) \delta \theta dV = 0. \]  
(14)

where \( \delta u_i \) is the virtual displacement. Integrating parts and the divergence theorem are used to obtain the weak formulations. The given equations are
\[-\int_V \sigma_{ij} \delta e_{ij} \, dV - \int_V \sigma_{ij} n_j \delta u_i \, dA + \int_V (F_i - \rho \delta \rho \delta u_i) \, dV = 0, \tag{15}\]

\[\int_T \delta \theta \, dV - \int_V q_i \delta \theta_i \, dV + \int_V q_i n_i \delta \theta_i \, dA = 0. \tag{16}\]

Substitute Equation (3) to (11) into Equations (15) and (16), to obtain the finite element formulations

\[\int_V \delta e_{ij} C_{ijkl} e_{kl} \, dV - \int_V \delta e_{ij} \beta_j \theta \, dV + \int_V \delta u_i \rho \delta \theta \, dV = \int_V \sigma_{ij} n_j \delta u_i \, dA + \int_V F_i \delta u_i \, dV, \tag{17}\]

\[T_0 \int_V \beta_j \delta e_{ij} \delta \theta_i \, dV + \int_V c_{ij} \delta \theta_i \delta \theta_i \, dV + \int_V \delta \theta_i k_i \theta_i \, dV = -\int_V q_i n_i \delta \theta_i \, dA. \tag{18}\]

In the finite element formulations, the displacement, temperature, strain fields, and temperature gradients are described by using shape functions as

\[u = H_u U, \quad \theta = H_{\theta} \Theta, \quad \epsilon = B_u U, \quad \theta_i = B_{\theta} \Theta, \tag{19}\]

where \(H_u\) and \(H_{\theta}\) are the displacement and temperature interpolation matrices, \(B_u\) and \(B_{\theta}\) are the strain-displacement matrix and the temperature-gradient interpolation matrix. \(U\) and \(\Theta\) are the displacement and temperature vectors. Substitute Eq. (19) into Eqs. (17) and (18) to obtain the weak formulation of the coupled equations in matrix form as

\[M \delta \dot{\theta} + K_u U = F, \tag{20}\]

\[C_{\theta \theta} \delta \theta + C_{u u} \delta \theta + K_u \Theta = Q, \tag{21}\]

where

\[M = \int_V H_u^T \rho H_u \, dV, \quad K_u = \int_V B_u^T \beta H \, dV, \quad K_{\theta \theta} = \int_V B_u^T C_{\theta u} B_u \, dV, \quad F = \int_T H_u^T \delta \rho \delta u \, dA + \int_T \tau \, dV, \quad C_{\theta \theta} = T_0 \int_V \beta \delta \theta \, dV, \quad C_{u u} = \int_V H_u^T C_{\theta u} H \, dV, \quad K_{\theta \theta} = \int_V B_u^T C_{\theta u} B_u \, dV, \quad Q = -\int_T H_u^T \delta Q \, dA.

**NUMERICAL SIMULATION**

**Numerical Implementations**

The Linux-based workstation is used for the finite element analysis of guided waves. The workstation have 16 quad core CPUs at 2.53 GHz, total 32GB memories and a 2 TB hard disk drive. Geometric parameters of the plate for simulations are presented in Fig. 1(a) : thickness T = 6 mm, center to end F = 152.4 mm and outside diameter OD = 114.3 mm. In Fig. 1(b), the schematic model of the methodology, called the partitioning method as in , is shown; the entire region is divided into two sub-region called the thermo-mechanical and the mechanical sub-regions. As shown in Fig. 1(b), the red region is thermo-mechanical sub-region where the temperature caused by laser pulse applied. While the thermo-mechanical sub-region has four degrees of freedom, three degrees of freedom are considered except one degree about the temperature in the mechanical sub-region as shown green region in Fig. 1(b).
Fig. 1. Simulation conditions of the laser excitation and detection of Lamb waves in a stainless a bent pipe: 
(a) geometric properties, (b) the partitioning scheme for numerical efficiency

One laser source point is shown in Fig. 2.(a) to analyze the output signal. The point one is laser input at one degree and the sensing point is the point two as shown in Fig. 2.(b). The distance between the point one and the point two is 328.27 mm. The mesh size of the stainless bent pipe varies because the pipe shape is not uniform. Therefore each mesh size is different as shown Fig. 2.(b). The one element shape at the outside of the bent pipe is almost the cubic, but the one element shape at the inside of the bent pipe is almost rectangular. Strictly speaking, those are not the cubic shape and the rectangular shape but the bent rectangular shape. The entire bent pipe consists of 706,320 elements.

Fig .2. Basic schematic of the bent pipe : (a) The position of the input points, (b) each mesh size and the output point
Signal analysis

Material properties of the stainless steel employed in present simulation are presented in Table 1. The time steps in computations are set as 0.5 ns for first 40 steps and 60ns for the rest. The laser pulse delivers 10 mJ energy for the pulse duration of 10ns at the point. Thermo-physical properties of the stainless are considered to obtain the power density of the laser source, so the input power density is 1.69 W / m² after consideration of the material properties.

Table 1. Material properties of the stainless employed for numerical simulations

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>193</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.237</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>8027</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (1/K)</td>
<td>2.31 x10⁻⁵</td>
</tr>
<tr>
<td>Specific heat (J/kg•K)</td>
<td>223.95</td>
</tr>
<tr>
<td>Thermal conductivity (W/m•K)</td>
<td>926.67</td>
</tr>
<tr>
<td>Lame’s first constant λ (GPa)</td>
<td>70.3</td>
</tr>
<tr>
<td>Lame’s second constant μ (GPa)</td>
<td>78.01</td>
</tr>
</tbody>
</table>

Finally, three dimensional Lamb wave propagation induced by laser pulses in bent pipe is observed in Fig. 3. (a). There are 3 degrees of freedom on displacement and the Fig. 3.(a) shows property of Lamb wave propagation on z-direction which is one of 3 degrees to confirm the normal displacement. Also, Fig. 3.(b) is the theoretical results in an aluminum plate by Shi et al.. Fig. 3.(a) is compared with Fig. 3.(b) because it is hard to solve analytical problem on bent pipe displacement by Lamb waves. Similarly, The Fig. 3.(a) and (b) show three type waves. One is longitudinal wave, another is transvers wave and the other is Rayleigh wave. The velocities of each wave from the numerical simulation are presented in Table 2. The velocities obtained by the numerical simulation are similar to theoretical wave speed from Eq. (22) and the velocities, from Eq. (22), of waves including all mode shape are shown in Table 2.

Table 2. Comparison between the theoretical and numerical results

<table>
<thead>
<tr>
<th>Wave type</th>
<th>Velocity from theoretical equations (m/s)</th>
<th>Velocity from numerical results (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal wave</td>
<td>5309.9</td>
<td>5320.5</td>
</tr>
<tr>
<td>Transvers wave</td>
<td>3117.47</td>
<td>3126.4</td>
</tr>
<tr>
<td>Rayleigh wave</td>
<td>2861.52</td>
<td>2829.9</td>
</tr>
</tbody>
</table>

\[
c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_T = \sqrt{\frac{\mu}{\rho}}, \quad c_R = c_T \left( \frac{0.87 + 1.12\nu}{1 + \nu} \right)
\] (22)
compare (a) predicted Lamb wave signals in an aluminum plate by Shi et al., with (b) simulated laser ultrasonic Lamb wave signal in a bent pipe

The three vertical arrows shown in Fig. 3 correspond to the longitudinal (L), transverse (T) and Rayleigh (R) waves, respectively. The results in Fig. 3 show raw data unfiltered and containing all the modes such the A and S modes. Nevertheless, the points of L-, T- and R waves are clearly noticeable. Also, the arrival times are similar to the theoretical values.

CONCLUSIONS

In this paper, the partitioning method that divides entire finite element region into the thermo-mechanical region and the mechanical region is employed to predict three-dimensional wave propagation in the bent pipe. The numerical results obtained from this method show a good agreement with the theoretical results. Also, computational efficiency is improved significantly without decrease in accuracy.

To assess wall-thinning in the bent pipe, we will make wall-thinning mesh in the bent pipe and compare the intact case with damaged case. In addition, conduct parameter study on various material properties is conducted and the wave signals induced by the laser source in the bent pipe are analyzed. Finally, we will establish reliability on noncontact NDT (nondestructive testing) by using the laser source and develop source code to improve the efficiency of the numerical calculation.

REFERENCES