Finite Element Analysis of Guided Waves by Noncontact Laser Excitation

Hyeong Uk Lim, Young Woo Ko and Jung-Wuk Hong
Department of Civil and Environmental Engineering,
Korea Advanced Institute of Science and Technology, Daejeon 305-701, Republic of Korea

ABSTRACT

This paper studies the generation of ultrasonic waves in plate-like structures. Here, the incidence of ultrasonic waves in the structures occurs due to the thermal expansion induced by laser pulses. For that reason, proposed is the methodology of the simulation that implements temperature profile in order to simulate the thermal expansion, using the finite element codes. After the simulation, through parametric analysis that performs examination of relationships between different parameters such as elastic modulus, Poisson ratio and density, the propagation of Lamb wave is investigated. Also, the thermo-mechanical response is observed during and after the thermal expansion. The feasibility of the proposed methodology is verified, through the comparison of the experiment and the numerical results.

1. INTRODUCTION

There have been many disasters in civil infrastructures including chemical plants and nuclear power plants. Since these terrible disasters are a crucial issue for humankind, many researchers make an effort to reduce accident that might happen in civil infrastructures. Existing evaluating techniques such as x-ray and exploration robot have some problems which cause interruption during the operation. Hence, we need to conduct researches on all time monitoring system to evaluate civil infrastructures. Especially, studies of using Lamb waves are critical to assess the performance of in-service structures and to maintain integrity of civil infrastructure [1]. In addition, Lamb waves contains much information over the thickness of the plate such as notch size and location [2]. Therefore, Lamb waves have potentials to evaluate pipe lines and plate-like structures [3].

Researchers make full use of methods of using Lamb waves all over engineering branch. Lee et al. [4] proposed method for rockbolt integrity inside tunnel using Fourier and wavelet transform. Hosseini and Ulrich [5] presented a method for numerical simulation of Lamb wave propagation in honeycomb sandwich panels. Lamb waves have also been employed for various defects such as holes and damages in carbon fibre composite. Lee et al. [6] presented method of monitoring pipe line using optical fiber-guided laser ultrasonic. Lamb waves have used to evaluate damage and condition at soil structure, composite material and structural health monitoring as above.

Apart from these, Lamb waves can be utilized as source of non-contact techniques. Many researchers make an effort to develop damage detection method to monitor the structures by using Lamb waves. Pavlopoulou et al. [7] illustrates applicability on postprocessing of guided ultrasonic waves for assessing the condition of structural health monitoring applications by experimenting on an aluminum repaired panel and carbon fibre-epoxy composite laminates. Additionally, Habib et al. [8] compare ultrasonic Lamb wave approach and surface mounted crack sensor approach for structural health monitoring of bounded composite repairs. Some researchers investigate application technique using piezoelectric transducers to estimate crack location and size and simulation numerical method [5, 9]. Like this, there are many non-contact technique researches using Lamb wave for evaluating rectangular plate, pipe and even thick plate [6, 10-12].

It is known that there are numerical analysis as well as experiment on Lamb waves transfer at thin plate and pipe for estimating defect. In this paper, we describe finite element analysis of guided waves by non-contact laser excitation. We observe the characteristics of three-dimensional wave propagation through experiment and simulation. For more efficient numerical calculation than previous research [13], the entire plate area is divided into the thermo-mechanical region and the mechanical region. The temperature changes in the thermo-mechanical region exited by the laser source while the temperature does not change at mechanical region. The
mechanical region has only material property except thermo-property. That means thermo-mechanical region has four degree of freedom, four independent factor such as x, y, z coordinate and thermo-factor, and mechanical region has three degree of freedom, three independent factor such as x, y, z, coordinate except thermo-factor. The goal of these all works is to increase efficiency and accuracy of calculation. It is the objective of this study to analyze wave patterns and compare arrival time of longitudinal, transverse, Rayleigh waves in plates generated by laser source.

2. THEORY

2.1 Finite element formulation for thermo-elastic analysis.

The finite element method is used for a wide range of engineering problem [3]. It has studied over the past several decades to use wave propagation in solid media. We have to consider the coupled thermo-elasticity to model Lamb waves excited by laser pulse. The equation of motion in local form and the local balance equation of entropy [14] are written, respectively as

\[ \sigma_{ij} + f_i = \rho \ddot{u}_i, \]  
\[ T_0 \dot{s} = -q_{ij}, \]  
where \( \sigma_{ij} \) are stresses, \( f_i \) are body forces, \( \rho \) is the density, and \( u_i \) are displacements. Also, \( T_0 \) is the equilibrium temperature, \( \dot{s} \) is the entropy rate per unit volume, and \( q_i \) is the thermal flux. For a well-posed boundary value problem, both mechanical and thermal boundary conditions must be satisfied. The boundary \( \Gamma \) of a domain can be decomposed as \( \Gamma = \Gamma_a \cup \Gamma_s = \Gamma_q \cup \Gamma_d \). The traction boundary condition \( \Gamma_a \) and the displacement boundary condition \( \Gamma_s \) are given as

\[ \sigma_{ij} n_j = \sigma_{ij} \text{ on } \Gamma_a, \]  
\[ u_i = u_i \text{ on } \Gamma_s, \]
where \( \sigma_{ij} \) are the prescribed surface tractions, and \( u_i \) are the prescribed displacements. The heat flux boundary \( \Gamma_a \) and the temperature boundary \( \Gamma_s \) are specified as

\[ q_{ij} n_j = \bar{Q} \text{ on } \Gamma_q, \]  
\[ \theta = \bar{\theta} \text{ on } \Gamma_d, \]  
where \( \bar{Q} \) and \( \bar{\theta} \) are the prescribed heat flux and temperature specified at thermal boundaries. The constitutive equations are stated as

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} - \beta_{ij} \varepsilon_{ij}, \]  
\[ s = \beta_{ij} \varepsilon_{ij} + \frac{eE}{T_0} \theta, \]  
where \( C_{ijkl} \) is the elasticity tensor, \( \varepsilon_{kl} \) is strain tensor, \( eE \) is the specific capacity, and \( \beta_{ij} \) is the thermo-mechanical coupling tensor which is of form for isotropic materials as

\[ \beta = \begin{bmatrix} 3k\alpha & 0 & 0 \\ 0 & 3k\alpha & 0 \\ 0 & 0 & 3k\alpha \end{bmatrix}. \]
where \( \kappa \) and \( \alpha \) denote the bulk modulus and the coefficient of thermal expansion, respectively. The strain-displacement relation is expressed as \( \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \), and Fourier’s heat conduction law, which states the relation between the heat flux and the temperature gradient, is given by

\[
q_i = -k_y \theta_j .
\]

where \( k_y \) is the thermal conductivity tensor as

\[
k = \begin{bmatrix}
k_1 & 0 & 0 \\
0 & k_2 & 0 \\
0 & 0 & k_3
\end{bmatrix}.
\]

Applying the principle of virtual work and the principle of virtual temperature to Equations [1] and [2], respectively, we have

\[
\int_V (\sigma_{ij,j} + f_i - \rho u_i) \delta u_i dV = 0 ,
\]

\[
\int_V (T_0 \delta \theta + q_i) \delta \theta dV = 0 .
\]

After integrating by parts and applying the divergence theorem, the weak formulations are obtained as

\[
-\int_V \sigma_{ij} \delta \varepsilon_{ij} dV - \int_V \sigma_{ij} n_i \delta u_i dA + \int_V (f_i - \rho u_i) \delta u_i dV = 0 ,
\]

\[
\int_V T_0 \delta \theta \delta \theta dV - \int_V q_i \delta \theta_i dV + \int_V q_i n_i \delta \theta dA = 0 .
\]

Substituting Equation [3] to [10] into Equations [14] and [15], we therefore have the results

\[
\int_V \varepsilon_{ij} C_{ij} \varepsilon_{ij} dV - \int_V \varepsilon_{ij} \beta_i \delta \theta dV + \int_V \varepsilon_{ij} n_i \delta u_i dA + \int_V f_i \delta u_i dV ,
\]

\[
T_0 \int_V \beta_i \varepsilon_{ij} \delta \theta dV + \int_V C_{ij} \delta \theta \delta \theta dV + \int_V \delta \theta_i k_i \delta \theta_i dV = -\int_V q_i n_i \delta \theta dA .
\]

In the finite element formulation, the displacement, temperature, strain fields, and temperature gradients are described by using shape functions as

\[
u = H_u U, \quad \theta = H_\theta \Theta, \quad \varepsilon = B_u U, \quad \theta_i = B_\theta \Theta ,
\]

where \( H_u \) and \( H_\theta \) are the displacement and temperature interpolation matrices, \( B_u \) and \( B_\theta \) are the strain-displacement matrix and the temperature-gradient interpolation matrix. \( U \) and \( \Theta \) are the displacement and temperature vectors. Substituting Eq. [18] into Eqs. [16] and [17], we can obtain the weak formulation of the coupled equations in matrix form as

\[
M \dot{U} + K_{uu} U + U = F ,
\]

\[
C_{uu} \dot{U} + C_{u\theta} \dot{\Theta} + K_{u\theta} \Theta = Q .
\]

where

\[
M = \int_V H_u^T \rho H_u dV , \quad K_{uu} = -\int_V B_u^T \beta H_u dV , \quad K_{uu} = \int_V B_u^T C B_u dV , \quad F = \int_V H_u^T \delta dA + \int_V H_u^T f dV ,
\]

\[
C_{uu} = T_0 \int_V H_u^T \beta B_u dV , \quad C_{u\theta} = \int_V H_u^T \varepsilon E H_u dV , \quad K_{u\theta} = \int_V B_u^T C B_u dV , \quad Q = -\int_V H_\theta^T \delta dA .
\]
3. NUMERICAL EXAMPLES

3.1 Numerical implementation

For a finite element analysis of guided waves, a Finite Element Analysis Program (FEAP), aimed at for research and educational use [15], is utilized under the Linux-based workstation. The workstation used has 16 quad core CPUs at 2.53GHz, total 32GB memories and a 2TB hard disk drive, operated in Ubuntu 11.10 version. Geometric parameters of the plate for simulations are presented in Figure 1(a): width \( w = 300 \text{ mm} \), length \( l = 300 \text{ mm} \) and thickness \( t = 2.0 \text{ mm} \). The distance between the laser source point and the sensing point is \( x = 133 \text{ mm} \). The mesh size of the aluminum plate is set to \( 1.25 \text{ mm} \times 1.00 \text{ mm} \times 0.80 \text{ mm} \), which ends up with 288000 elements. In Figure 1(b), the schematic model of the methodology, called the partitioning method as in [16], is shown; the entire region is divided into two sub-area, namely, the thermomechanical subregion and the mechanical subregion. Concerns in determining the area of the thermomechanical subregion inextricably rely on how the temperature caused by laser pulses propagates. Therefore, in the simulations it is assumed that the area of the thermomechanical subregion corresponds to the area of \( 100 \text{ mm} \times 100 \text{ mm} \times 3.2 \text{ mm} \) as shown in Figure 1(b). In mechanical subregion, three degree of freedom is considered, while in thermomechanical subregion four degree of freedom are considered.

![Figure 1](image_url)

**Figure 1.** Simulation conditions of the laser excitation and detection of Lamb waves in an aluminum plate: (a) geometric properties, (b) the partitioning scheme for numerical efficiency.

3.2 Verification of the simulation

From the theoretical results obtained in an aluminum plate by Yijun et al. [17], the comparative study is performed. Material properties of the aluminum employed in present simulation are presented in Table 1. The time steps in computations set as 1ns for first 20 steps and 50ns for remnants. The duration of laser pulse is very short (8ns) and the response for short duration is similar to a Dirac delta function. Source diameter of laser pulses is \( a = 0.5 \text{ mm} \). Finally, three dimensional Lamb wave propagation induced by laser pulses in thin aluminum plate is observed in Figure 2(b). In verification study, signal obtained from simulations show a good fidelity to the theoretical results.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>70.2</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.345</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2769</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (1/K)</td>
<td>( 2.31 \times 10^{-5} )</td>
</tr>
<tr>
<td>Specific heat (J/kg·K)</td>
<td>223.95</td>
</tr>
<tr>
<td>Thermal conductivity (W/m·K)</td>
<td>926.67</td>
</tr>
</tbody>
</table>

Table 1. Material properties of the aluminum employed for numerical simulations.
The arrival times of bulk waves are \( t = 56.5 \) sec (the longitudinal wave), \( t = 96.2 \) sec (the transverse wave), and \( t = 103.9 \) sec (Rayleigh wave). Velocity of each wave is calculated directly from the numerical result shown in Figure 2.(b) to the equation \( v = \frac{d}{t} \) where \( v \) is velocity, \( d \) is distance between the laser source and the sensing point, and \( t \) is elapsed time. Measured velocity of each wave are 6315.3 m/s (the longitudinal wave), 3243.9 m/s (the transverse wave), and 3036.5 m/s (Rayleigh wave), respectively. Meanwhile theoretical wave velocity for each wave in present aluminum plate is 6310.8 m/s (the longitudinal wave), 3070.1 m/s (the transverse wave), and 2890.5 m/s (Rayleigh wave), which are in congruence with numerical values.

\[ r \frac{\lambda + 2\mu}{\rho}, \quad c_T = \sqrt{\frac{\mu}{\rho}}, \quad c_R = c_S \left( \frac{0.87 + 1.12v}{1 + \nu} \right) \]  

Figure 2. Comparison between the predicted result and the numerical result: (a) predicted laser ultrasonic Lamb wave signal by Shi et al. [17] and (b) simulated laser ultrasonic Lamb wave signal in an aluminum plate.

3.3 Case studies

Experiment on ultrasonic wave propagation caused by laser pulses in a specimen such as an aluminum plate has been significantly executed due to versatile use of aluminum material. Hence, we refocus attention on studies of characteristics of Lamb wave propagation under alteration of material properties such as elastic modulus, Poisson ratio and density. In this studies, observed are changes of each wave velocities and shape of signals, which help us to understand a rudimentary feature of waves in the plate. After the observation, the comparison between the numerical results and the theoretical results is performed to aid in the selection of material properties of the plate.

Different elastic modulus

To observe the effect of the different elastic moduli in three dimensional wave propagation, the elastic moduli are set to 50 GPa, 70 GPa, and 90 GPa with the fixed values of other parameters. Accordingly, theoretical velocities of longitudinal, transverse, and Rayleigh wave are respectively (a) 5326 m/s, 6302 m/s, and 7146 m/s; (b) 2591 m/s, 3067 m/s, and 3476 m/s; (c) 2439 m/s, 2886 m/s, and 3273 m/s. Formulations of theoretical velocities of longitudinal (\( c_L \)), transverse (\( c_T \)), and Rayleigh wave (\( c_R \)) are shown as

\[ c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_T = \sqrt{\frac{\mu}{\rho}}, \quad c_R = c_S \left( \frac{0.87 + 1.12v}{1 + \nu} \right) \]  

where \( \lambda \) is Lame’s first parameter, \( \mu \) is the shear modulus, \( \nu \) is Poisson’s ratio. With the large elastic modulus, the wave velocities calculated directly from the graph in Figure 3 gets faster; (a) 5341.9 m/s, 6315.3 m/s, and 7157.5 m/s; (b) 2742.3 m/s, 3243.9 m/s, and 3674.0 m/s; (c) 2572.3 m/s, 2890.5 m/s, and 3445.6 m/s. In consequence, these results would provide us that linear relationship between elastic modulus and bulk wave velocity existed.
Figure 3. Lamb wave simulations for different elastic modulus: (a) 50 GPa, (b) 70 GPa, and (c) 90 GPa.

Different Density

Figure 4. shows the delay in wave propagation in the material under change of density. As density of material increase, arrival time of wave modes is somewhat delayed. Shape of wave in each case remains same through the modification in densities.
Figure 4. Lamb wave simulations for different density: (a) 2769 kg/m$^3$, (b) 3000 kg/m$^3$, and (c) 4000 kg/m$^3$. 
**Different Poisson ratio**

With the different Poisson ratio, the signals obtained shows changes of its amplitude as shown in Figure 5.

Table 2. Comparison of velocity between theoretical and numerical method.

<table>
<thead>
<tr>
<th>Poisson ratio</th>
<th>Type of wave</th>
<th>Theoretical velocity</th>
<th>Measured velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>Longitudinal wave</td>
<td>5507.8 m/s</td>
<td>5519 m/s</td>
</tr>
<tr>
<td></td>
<td>Transvers wave</td>
<td>3179.9 m/s</td>
<td>3174.2 m/s</td>
</tr>
<tr>
<td>0.3</td>
<td>Longitudinal wave</td>
<td>5833.6 m/s</td>
<td>5846 m/s</td>
</tr>
<tr>
<td></td>
<td>Transvers wave</td>
<td>3118.2 m/s</td>
<td>3197.12 m/s</td>
</tr>
</tbody>
</table>

Theoretical velocity is obtained from a theoretical approach Eq [21]. Table 2 shows theoretical velocity is similar to measured velocity. This result shows different pattern above. Changing modulus of elasticity and density, graph pattern is not changed, but velocity is only changed. However, graph pattern is changed trough Poisson ratio. It is estimated it is related to transvers property of materials, actually Poisson ratio is deeply related to transvers property of materials. It is necessary to solve theoretically wave equation and understand relation between wave equation and physical character.
4. SUMMARY AND CONCLUSIONS

In this paper, the partitioned method that divides entire finite element region into the thermo-mechanical region and the mechanical region is employed to predict three-dimensional wave propagation in the plate. The numerical result obtained from this method shows a good agreement with the experimental result. Also, computational efficiency is improved significantly without decrease in accuracy.

Through the parametric study about variables used in the simulation, the characteristics of the wave propagation in the plate are examined. It is found that the arrival times of wave forms in the plate are shortened as the elastic modulus of the plate increase. In contrast, the arrival times of wave forms are delayed when the density of the plate is increased. Poisson ratio of the plate affects the amplitude of wave forms.

Theoretical analysis of the wave propagation induced by laser pulses still requires further investigation but demonstrates the potential of assist in the study of characteristics of waves in the plate. The next step in this study is to apply the results found in this work to damage detection method for the structure health monitoring.

REFERENCES


